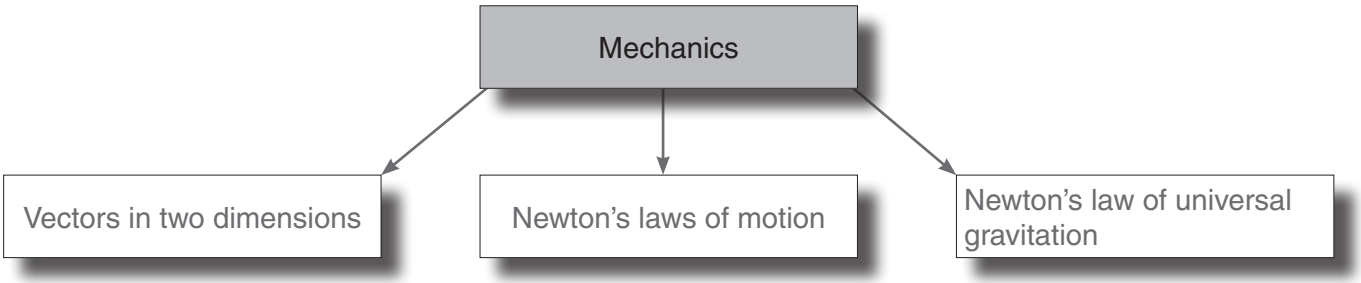
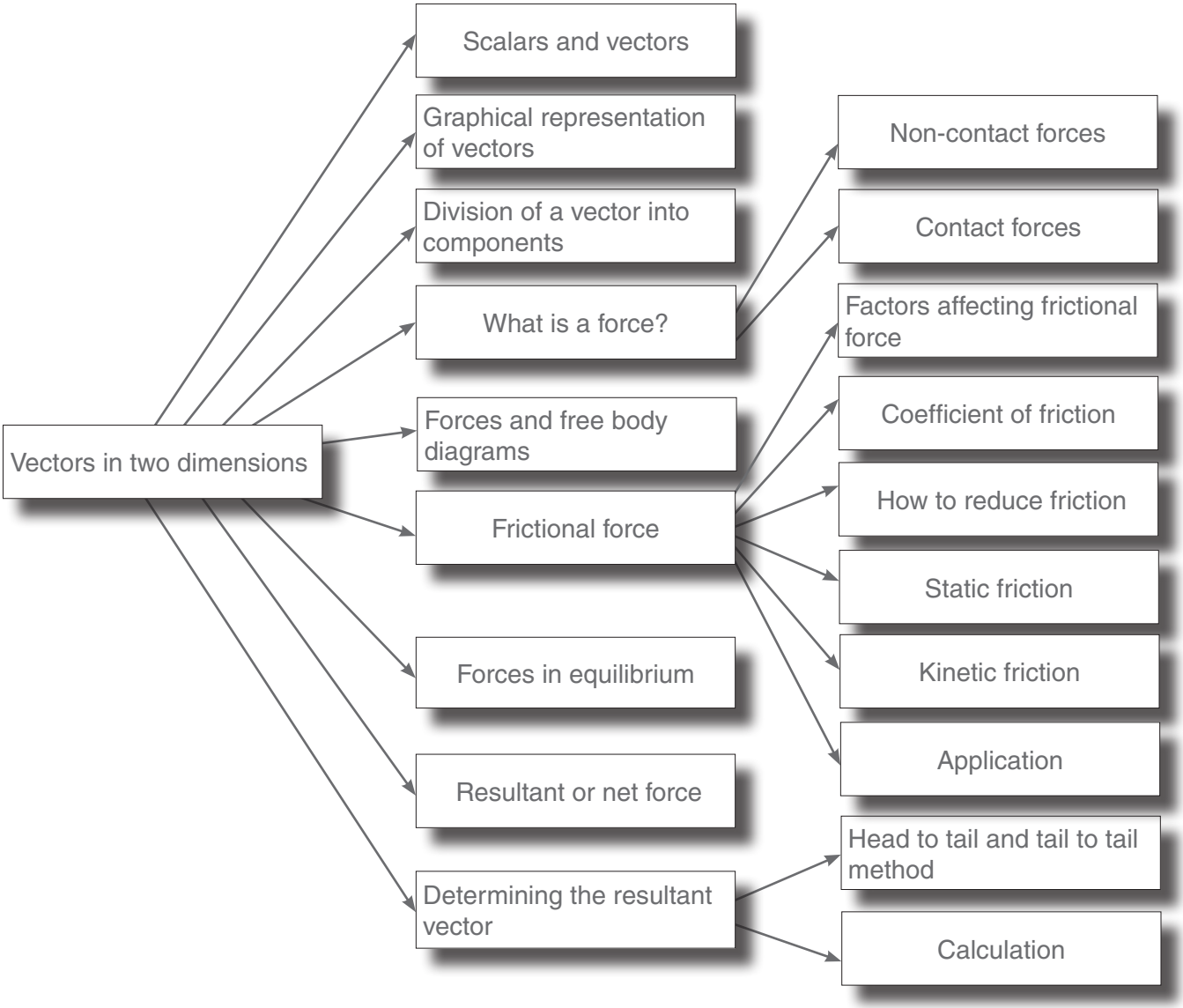
Masters for transparencies

5.1 Mechanics

KNOWLEDGE AREA: MECHANICS



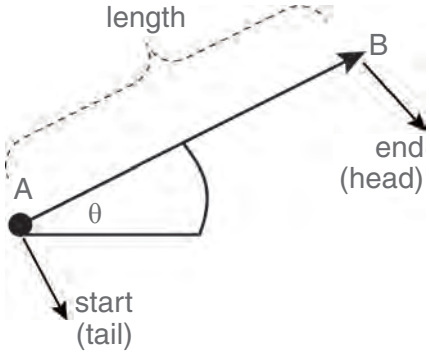
UNIT 1 VECTORS IN TWO DIMENSIONS



1.1 Scalars and vectors

Scalar	Vector
A physical quantity having magnitude and a unit, but not direction.	A physical quantity having magnitude, a unit and direction.
Examples: mass (6 kg); time (5 s); distance (2 m); speed (60 $m \cdot s^{-1}$); volume (20 m^3); wavelength (60 × 10^{-6} m); energy (200 J); work (240 J); power (1 200 W); temperature (273 K); electric current (2 A); electrical potential difference (12 V)	Examples: force (6 N upward); weight (340 N downward); displacement (40 m west); velocity (5 $m \cdot s^{-1}$ direction 30°); acceleration (4 $m \cdot s^{-2}$ left); momentum (5 $kg \cdot m \cdot s^{-1}$ east); impulse (6 N·s west)

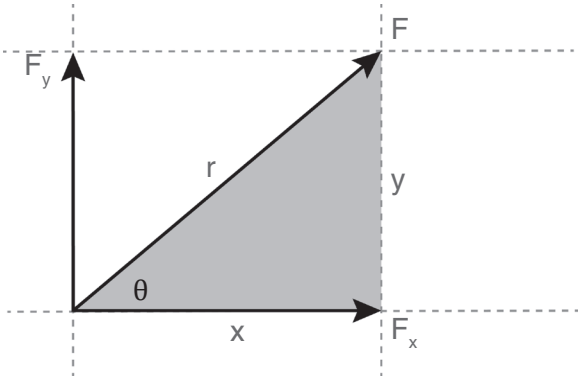
1.2 Graphical representation of vectors



- Length of the arrow represents the magnitude of the vector.
- Arrowhead shows the direction of the vector.

1.3 Division of a vector into components

Components of a force exerted at an angle to the horizontal plane



The following ratios are valid for the shaded triangle:

$\cos\theta = \frac{x}{r} = \frac{F_x}{F}$ AND $\sin\theta = \frac{y}{r} = \frac{F_y}{F}$

Horizontal component = $F_x = F\cos\theta$ → applied force; angle with regards to x-axis

Vertical component = $F_y = F\sin\theta$ → angle with regards to x-axis; applied force

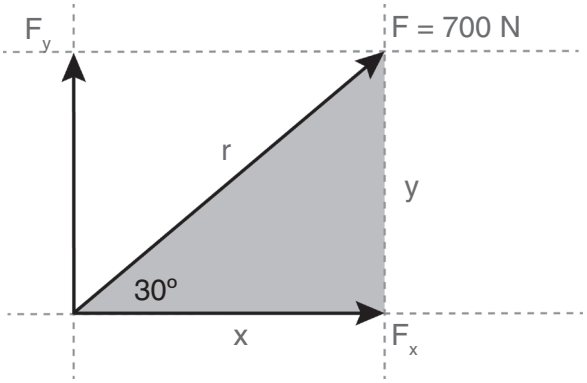
Examples

Charl pulls a grass roller over a horizontal lawn with a force of 700 N.
The handle of the roller makes an angle of 30° with the horizontal plane.

1. Calculate the x-component (horizontal) F_x.
2. Calculate the y-component (vertical) F_y for the force exerted by Charl.
3. Charl now pushes the roller over the lawn with the same force at the same angle. How do the components of the force change?
4. Is it better for the grass if the roller is pushed or pulled? Give a reason for your answer.

Solutions:

Draw a diagram, not necessarily to scale. Label each vector so that it is clear what it represents.



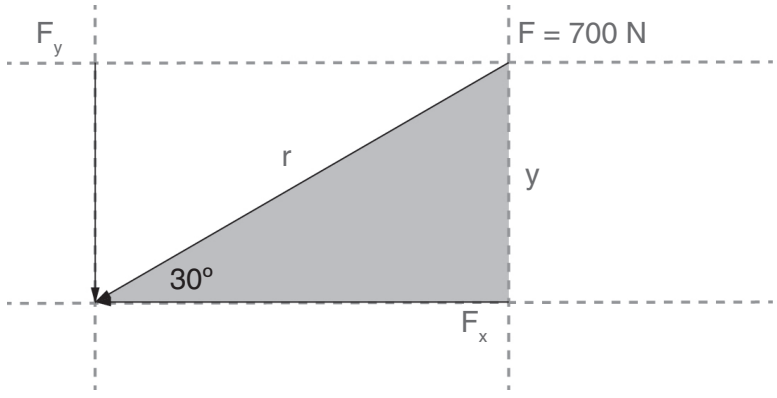
1. In the shaded triangle, F_x can be calculated using the cos function:

$$\cos 30° = \frac{x}{r} = \frac{F_x}{F}$$
$$\text{therefore: } F_x = F\cos 30°$$
$$F_x = 700\cos 30°$$
$$F_x = 606{,}22 \text{ N horizontal}$$

2. In the shaded triangle, F_y can be calculated using the sin of the angle:

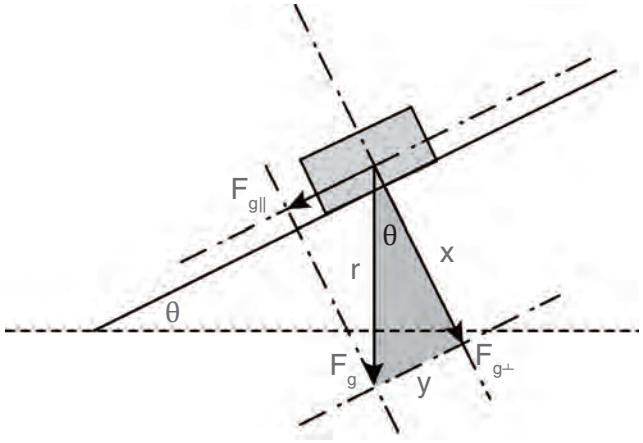
$$\sin 30° = \frac{y}{r} = \frac{F_y}{F}$$
$$\text{therefore: } F_y = F\sin 30°$$
$$F_y = 700\sin 30°$$
$$F_y = 350 \text{ N vertical}$$

3. Although the magnitude of the vectors remains the same, the directions change.



4. F_x is directed downward if the roller is pushed. Charl is therefore pushing the roller into the ground when he pushes it. Although it is harder to push the roller, it is more beneficial as it is flattening the ground with a greater force.

Components of a force acting at an angle to a slope



The following ratios are valid for the shaded triangle:

$$\cos\theta = \frac{x}{r} = \frac{F_{g\perp}}{F_g} \qquad \text{AND} \qquad \sin\theta = \frac{y}{r} = \frac{F_{g\parallel}}{F_g}$$

gravitational force

Perpendicular component $= F_{g\perp} = F_g \cos\theta$
Parallel component $= F_{g\parallel} = F_g \sin\theta$

angle with regard to horizontal

gravitational force

Examples

A car with a mass of 1 500 kg is parked on a slope of 30°.

1. Calculate the component of the weight of the car that is parallel to the slope.
2. Calculate the component of the weight of the car that is perpendicular to the slope.
3. When the car moves further up the road, the slope increases to 40°. How will the components of the weight change, respectively? Explain your answer.

Solutions:

First draw a labelled diagram.