Grade 11 Physics

Knowledge Area: Mechanics

Vectors in two dimensions

1.1 Scalars and vectors

Scalar: A physical quantity having magnitude and a unit, but not direction.

Scalar

Examples:

mass (6 kg); time (5 s); distance (2 m);
speed (60 m ▪ s^{-1}); volume (20 m^3);
wavelength (60 × 10^{-6} m);
energy (200 J); work (240 J);
power (1 200 W); temperature (273 K);
electric current (2 A);
electrical potential difference (12 V)

Vectors in two dimensions

Vector: A physical quantity having magnitude, a unit and direction.

Definition

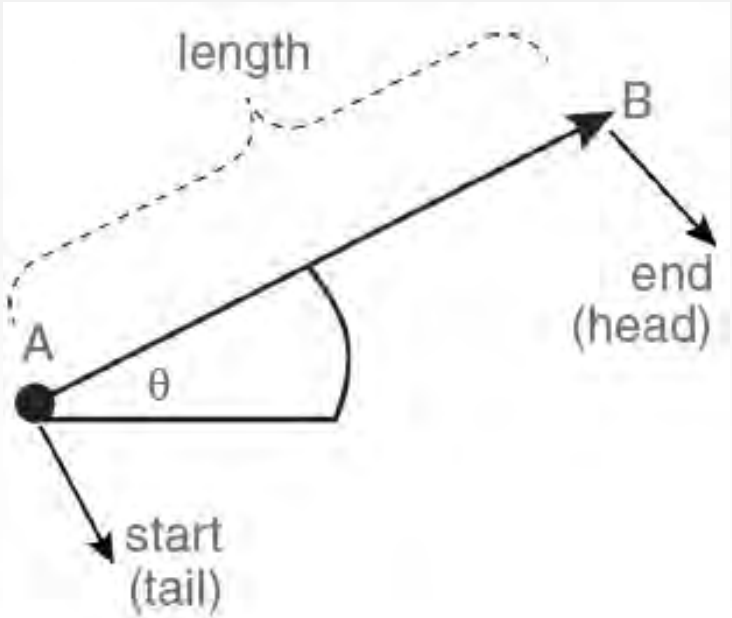
Vector

Examples:
force (6 N upward);
weight (340 N downward);
displacement (40 m west);
velocity (5 m ▪ s^{-1} direction 30°);
acceleration (4 m ▪ s^{-2} left);
momentum (5 kg ▪ m ▪ s^{-1} east);
impulse (6 N ▪ s west)

Vectors in two dimensions

1.2 Graphical representation of vectors

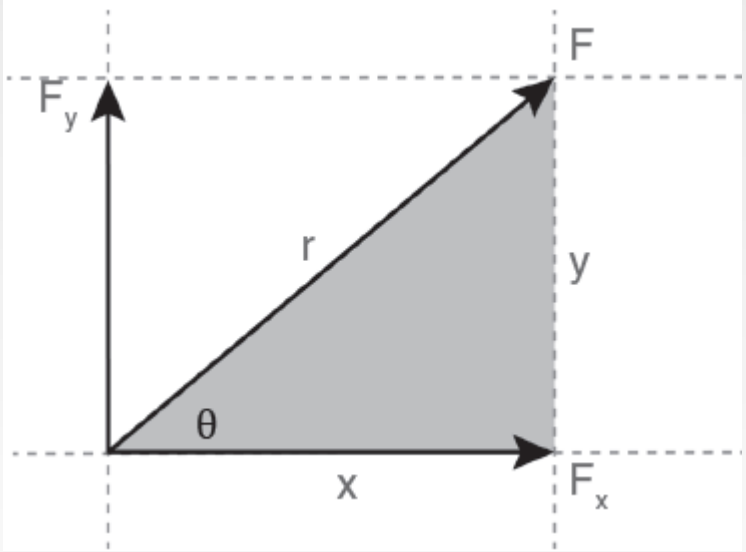
- Length of the arrow represents the magnitude of the vector.
- Arrowhead shows the direction of the vector.



Vectors in two dimensions

1.3 Division of a vector into components

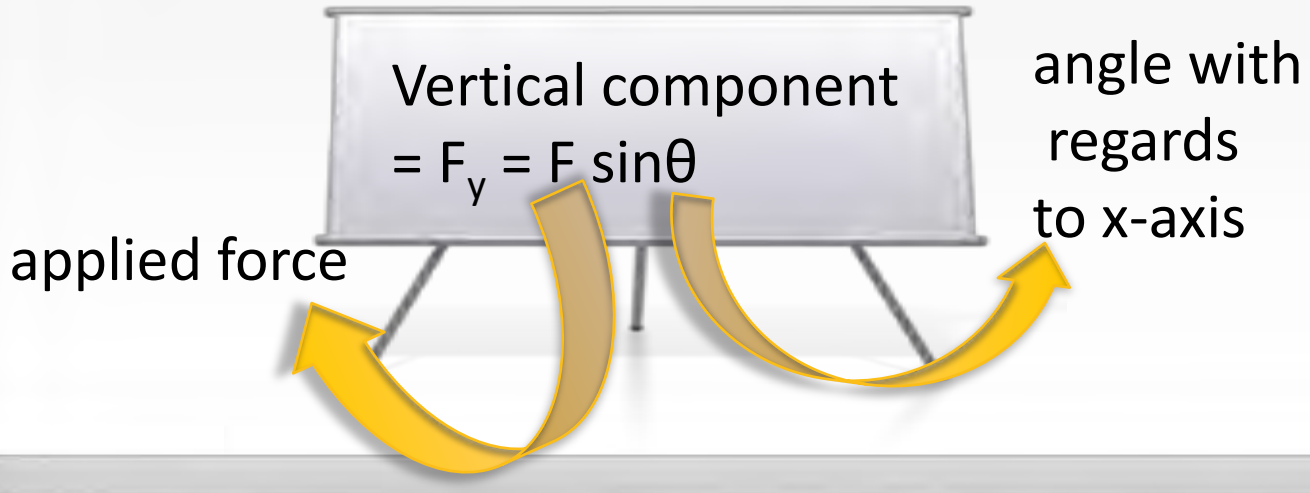
Components of a force exerted at an angle to the horizontal plane

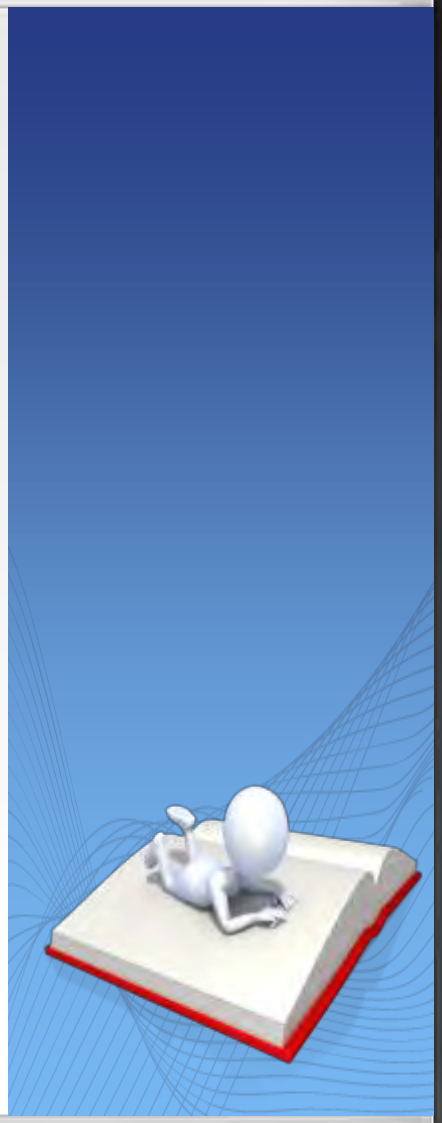


The following ratios are valid for the shaded triangle:

$$\cos\theta = \frac{x}{r} = \frac{F_x}{F} \quad \text{AND} \quad \sin\theta = \frac{y}{r} = \frac{F_y}{F}$$

Vectors in two dimensions





Vectors in two dimensions

Examples

Charl pulls a grass roller over a horizontal lawn with a force of 700 N.
The handle of the roller makes an angle of 30° with the horizontal plane.

1. Calculate the x-component (horizontal) F_x.
2. Calculate the y-component (vertical) F_y for the force exerted by Charl.
3. Charl now pushes the roller over the lawn with the same force at the same angle. How do the components of the force change?
4. Is it better for the grass if the roller is pushed or pulled? Give a reason for your answer.

Vectors in two dimensions

Solutions:

Draw a diagram, not necessarily to scale. Label each vector so that it is clear what it represents.

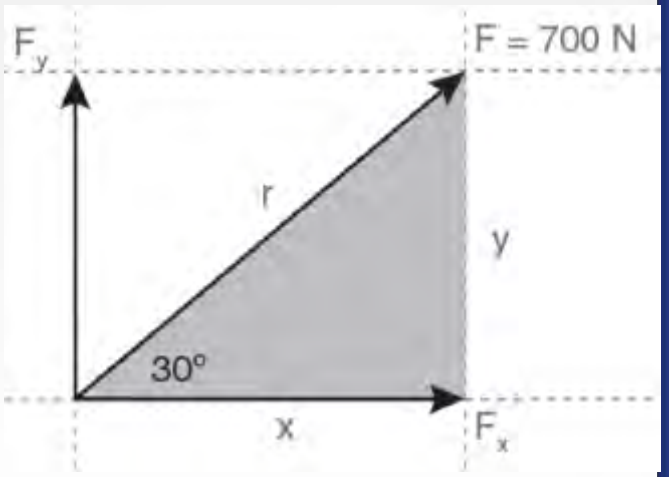
1. In the shaded triangle, Fx can be calculated using the cos function:

$$\cos 30° = \frac{x}{r} = \frac{F_x}{F}$$

therefore: $F_x = F \cos 30°$

$F_x = 700 \cos 30°$

$F_x = 606{,}22$ N, horizontal



Vectors in two dimensions

2. In the shaded triangle, Fy can be calculated using the sin of the angle:

$$\sin 30° = \frac{y}{r} = \frac{Fy}{F}$$

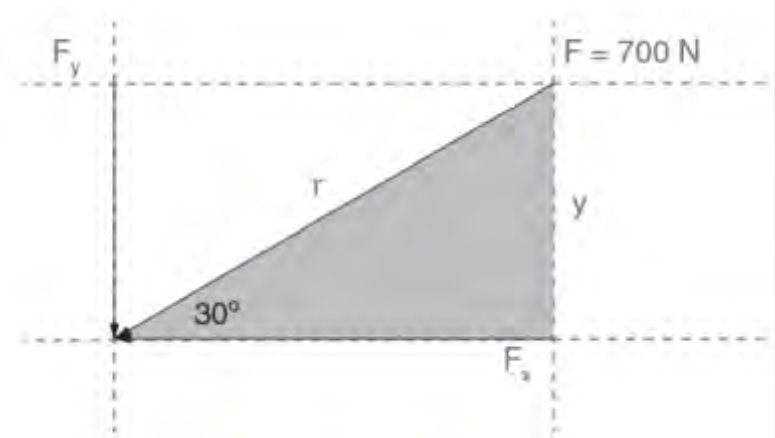
therefore: $F_y = F \sin 30°$

$F_y = 700 \sin 30°$

$F_y = 350$ N, vertical

Vectors in two dimensions

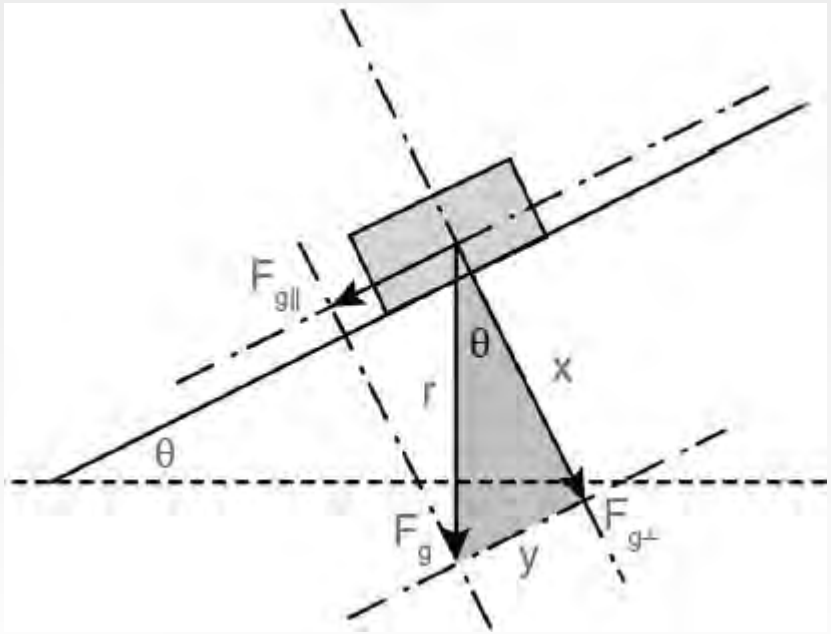
3. Although the magnitude of the vectors remains the same, the directions change.



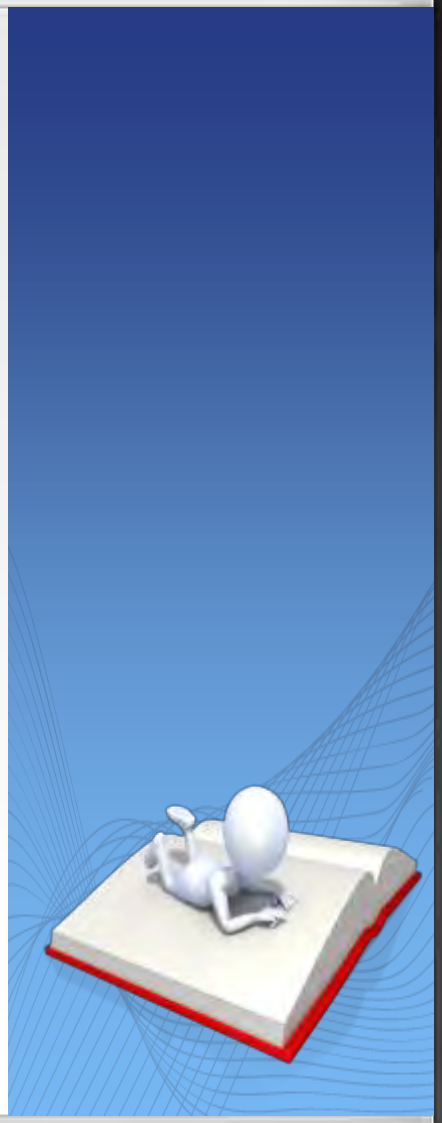
4. F_x is directed downward if the roller is pushed. Charl is therefore pushing the roller into the ground when he pushes it. Although it is harder to push the roller, it is more beneficial as it is flattening the ground with a greater force.

Vectors in two dimensions

Components of a force acting at an angle to a slope



The following ratios are valid for the shaded triangle:



Vectors in two dimensions

$$\cos\theta = \frac{x}{r} = \frac{F_{g\perp}}{F_g} \quad \text{AND} \quad \sin\theta = \frac{y}{r} = \frac{F_g}{F_g}$$

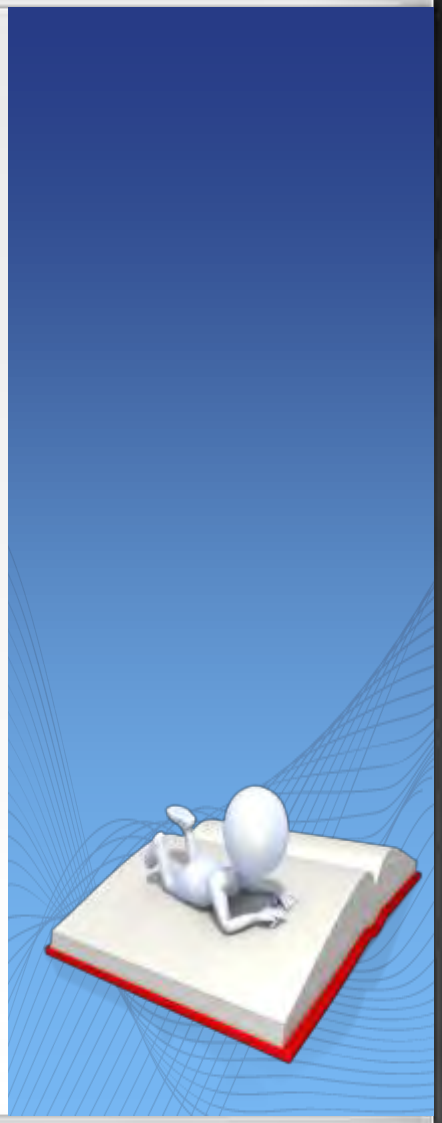
gravitational force

Perpendicular component
$= F_{g\perp} = F_g \cos\theta$

gravitational force

Parallel component
$= F_{g\parallel} = F_g \sin\theta$

angle with regard to horizontal

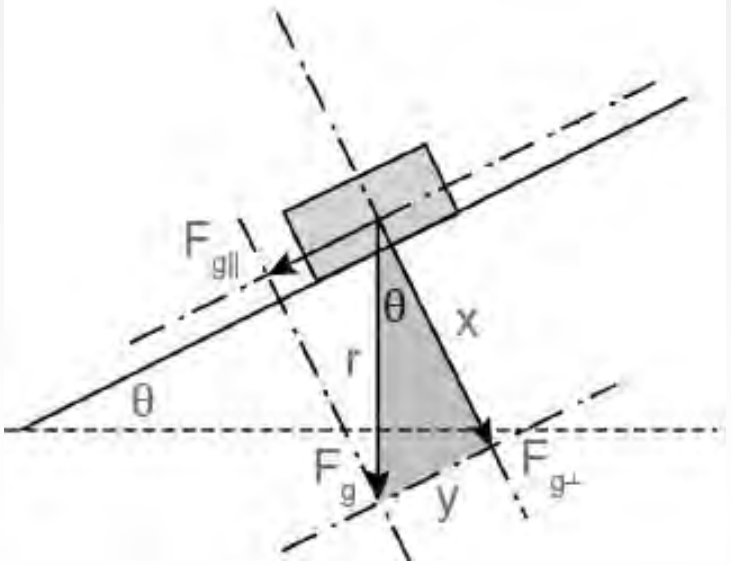


Vectors in two dimensions

Examples

A car with a mass of 1 500 kg is parked on a slope of 30°.

1. Calculate the component of the weight of the car that is parallel to the slope.
2. Calculate the component of the weight of the car that is perpendicular to the slope.
3. When the car moves further up the road, the slope increases to 40°. How will the components of the weight change, respectively? Explain your answer.



Solutions:

First draw a labelled diagram.

Vectors in two dimensions

1. For the shaded triangle, the parallel component is:

$F_g = F_g \sin\theta$

$= (1\,500 \times 9{,}8)\sin 30°$

$= 7\,350$ N, parallel to the level downwards.

2. For the shaded triangle the perpendicular component is:

$F_{g\perp} = F_g \cos\theta$

$= (1\,500 \times 9{,}8)\cos 30°$

$= 8\,487{,}05$ N, perpendicular to the level upwards.

Vectors in two dimensions

3. If the slope increases, the parallel component increases. The magnitude of the parallel component is directly proportional to sinθ. As θ increases, sinθ increases. Therefore F_g will increase as θ increases. If the slope increases, the perpendicular component will decrease. The magnitude of the perpendicular component is directly proportional to cosθ. As θ increases, cosθ decreases. Therefore as θ increases $F_{g\perp}$ decreases.